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International Journal of Heat and Mass Transfer 42 (1999) 3887–3899

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Similarity of transport processes in fluidized beds

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Received 30 July 1996

Abstract

The theory of similarity of heat and mass transfer processes in bubble fluidized beds is developed. It is based on the account for a main specific feature, which distinguishes a fluidized bed from other gas-solid systems, viz. a two-phase structure of the bed: the presence of bubble and emulsion phases. The existence of these phases determined the character of transport processes in the system in many respects and imparts a number of unique properties to the system that are widely used in engineering (isothermicity, high values of the coefficients of thermal conductivity and heat transfer, etc.). A system of dimensionless governing parameters which characterize local and integral similarity of fluidized beds is found. It is shown that *Fr* and *Ar* numbers can serve as basic dimensionless characteristics of bubble and emulsion phases. The employment of these numbers makes it possible to obtain generalized relations for calculating transport characteristics of fluidized beds which are distinguished by simplicity, physical and large universal nature.

1. Introduction

The development of the methods for calculating transport processes in bubble fluidized beds (FB) is, to a great extent, complicated by the absence of a rather rigorous theory a fluidized state of granular medium [1,2]. By virtue of this fact, different empirical and semiempirical investigation methods have acquired wide spread. In this situation, as is known [3], of especial value are the methods of the similarity theory which allow, on the one hand, the organization of rational experimental studies and, on the other, the retention of rather broad generalizations of the available experimental data that are applicable in engineering practice.

As is known [3], non-dimensionalization of differential equations describing these processes is a classical method for obtaining dimensionless determining variables (simplexes and complexes) which reflect similarity of these processes. As applied to such heterogeneous system as a FB these are the equations of balance of masses, momenta, and energies of gas and particles. This means was realized in [4–7] where the equations of balance of masses and momenta of phases obtained by Anderson and Jackson [8] were non-dimensionalized. Another means for obtaining dimensionless criteria of similarity which is widely used in practice is an application of the π-theorem of similarity theory [3] to the empirically found system of dimensional parameters determining the unknown quantity. A great number of dimensionless equations for calculating different characteristics of a FB were obtained by this (in many respects euristic) means [9–11]. It is obvious, that the first means is preferable due to its substantiated and universal nature. Nevertheless, it should be stated that this means did not give a system of dimensionless parameters which allow one to adequately reveal the laws of similarity in a FB. To our opinion, the problem is in the fact that the mentioned Andersen-Jacksson equations do not suit to such an analysis because they are written separately for gas and particles and do not reflect another important aspect of a two-phase nature of a FB, i.e. the presence of bubble and emulsion phases [1]. Just this type of a

0017-9310/99/$ - see front matter
PII: S0017-9310(99)00044-7

Nomenclature

Ar	Archimedes number, $gd^3\rho_f(\rho_s-\rho_f)/\mu_f^2$
c	specific heat capacity
D_f	coefficient of gas diffusion
D	bed diameter
D_t	tube diameter
D_b	gas bubble diameter
D_h	vertical size of a bubble, $D_h=0.7D_b$
D^h	coefficient of horizontal diffusion of particles
d	particle diameter
Fr_h	local Froude number, $(u-u_{mf})^2/gh$
Fr	Froude number, $(u-u_{mf})^2/gH_{mf}$
g	capacity acceleration
H_{mf}	bed height at $u=u_{mf}$
H	bed height
h	local height above a gas distributor
H_c	height of a separation zone
k	coefficient of mass transfer related to bubble unit surface
K	coefficient of effective vertical diffusion of particles
Nu	Nusselt number, $\alpha d/\lambda_f$
P	pressure
Pe_b	Pecklet number, $v_b D_b/2D_f$
Pr	Prandtl number, $c_f\mu_f/\lambda_f$
Re	Reynolds number, $ud\rho_f/\mu_f$
Re_t	Reynolds number, $uD_t\rho_f/\mu_f$
Re_{mf}	Reynolds number, $u_{mf}d\rho_f/\mu_f$
Re_b	Reynolds number, $v_b D_b\rho_f/2\mu_f$
Sc	Schmidt number, $\mu_f/\rho_f D_f$
Sh	Sherwood number, $kD_b/2D_f$
S_v, S_h	vertical and horizontal pitches of tubes
u	superficial gas velocity
u_{mf}	minimum fluidization velocity
u_b	absolute velocity of a bubble
v_b	relative velocity of a bubble, $v_b=u_b-(u-u_{mf})$
w_{su}	root-mean-square velocity of ejection of particles from a bed

Greek symbols

α	coefficient of heat transfer
β	coefficient of mass exchange between bubble and emulsion phases, related to bed unit volume
ε_{mf}	bed porosity at $u=u_{mf}$
ε	mean porosity of a bed
ε_b	concentration of bubbles
ε_p	porosity of a packing
μ_f	dynamic viscosity of gas
λ_f	gas thermal conductivity
ρ	density

Subscripts

b	bubble
cond	conductive
conv	convective
c–c	conductive-convective
f	gas

fb	freeboard
e	emulsion
su	surface of a fluidized bed
s	particles

two-phase nature distinguishes a FB from other disperse systems (a stationary infiltrated bed, a falling dense bed, pneumotransport of particles, etc.). The presence of these phases determines the character of transport processes in a FB in many respects and, as is known, imparts a number of unique properties to its (high isothermicity, large values of the diffusion and heat transfer) coefficients.

2. Governing dimensionless parameters

The technique for obtaining similarity criteria of transport processes in a FB that is based on the utilization of the π-theorem and account for a two-phase nature of the system: the presence of bubble and emulsion phases, is presented.

2.1. Bubble phase

The basic equation describing gas flow distribution over the phases in a FB (two-phase theory equation) has the form

$$u = mu_{\mathrm{mf}}(1-\varepsilon_{\mathrm{b}}) + u_{\mathrm{b}}\varepsilon_{\mathrm{b}} + nu_{\mathrm{mf}}\varepsilon_{\mathrm{b}}, \tag{1}$$

where mu_{mf} is the velocity of gas in the emulsion phase; nu_{mf} is the velocity of the through (with respect to a bubble) gas flow. Introducing a specific gas flow in the form of bubbles (a 'visible' bubble flow) Eq. (1) is obtained in the form

$$G_{\mathrm{b}} = u_{\mathrm{b}}\varepsilon_{\mathrm{b}} = u - u_{\mathrm{mf}}(m(1-\varepsilon_{\mathrm{b}}) + n\varepsilon_{\mathrm{b}}). \tag{2}$$

As experience shows, gas distribution over the phases with an accuracy acceptable for practice can be described by simplified versions of (2);

1. fine particles ($d < 1$ mm)

   $$m = n = 1, \quad G_{\mathrm{b}} = u - u_{\mathrm{mf}} \tag{3}$$

   is the equation of the 'ideal' two-phase theory by Toomey and Johnstone [12];
2. coarse particles ($d \geq 1$ mm)

   $$m = 1, \quad G_{\mathrm{b}} = u - u_{\mathrm{mf}}(1 + (n-1)\varepsilon_{\mathrm{b}}), \tag{4}$$

   is the equation first used in [13]. The quantity n which characterizes a gas flow through a bubble changes from 3 to 8 [13] and is rather a complex function of an excess gas velocity $(u - u_{\mathrm{mf}})$ and a bed height. Eliminating ε_{b} from (4), we can present this equation in the form

   $$G_{\mathrm{b}} = (u - u_{\mathrm{mf}})\frac{u_{\mathrm{b}}}{u_{\mathrm{b}} + (n-1)u_{\mathrm{mf}}}. \tag{5}$$

   Giving a determining role in the formation of a hydrodynamic flow pattern of gas and particles in a FB a 'visible' bubble flow, within the aims of the present study an important conclusion can be made on the expediency of using an excess gas velocity $u - u_{\mathrm{mf}}$ as a velocity scale typical of a bubble phase. A local size of a bubble, its velocity and other characteristics are, moreover, determined by the value of the current coordinate h, the diameter and the height of the bed that affect the coalescence of bubbles. As bubbles rise in the gravity field, the quantity g should be added to the governing parameters.

Any hydrodynamic characteristic of the bubble phase B (velocity, size, and concentration of bubbles in the bed, etc.) will, in a general case, be the function of the above parameters

$$B = f(u - u_{\mathrm{mf}}, H_{\mathrm{mf}}, D, g, h). \tag{6}$$

The employment of the π-theorem of the similarity theory makes it possible to write instead of (6) its dimensionless analogue

$$B' = f\left(Fr, \quad \frac{H_{\mathrm{mf}}}{D}, \quad \frac{h}{H_{\mathrm{mf}}}\right), \tag{7}$$

where $Fr = (u - u_{\mathrm{mf}})^2/gH_{\mathrm{mf}}$ is the Froude number characterizing the ratio of the kinetic energy of gas bubbles to the potential energy of bubble floating. Thus, the bubble phase is characterized by three governing dimensionless parameters

$$Fr, \frac{H_{\mathrm{mf}}}{D}, \frac{h}{H_{\mathrm{mf}}}. \tag{8}$$

2.2. Emulsion phase

One of the more principal statements of the two-phase theory of a FB, repeatedly confirmed by experiments, is the fact that the emulsion phase of a FB is in the state close to the minimum fluidization.

Consequently, in interparticle gaps the characteristic space and velocity scales are d and u_{mf}. The governing parameters of the emulsion phase are c_{f}, c_{s}, ρ_{f}, ρ_{s}, μ_{f}, D_{f}, and λ_{f}. It can be written for an arbitrary characteristic of the emulsion phase E (coefficients of interphase heat and mass transfer, heat exchange between emulsion and the surface, etc.)

$$E = \varphi(d, u_{\mathrm{mf}}, c_{\mathrm{f}}, c_{\mathrm{s}}, \rho_{\mathrm{f}}, \rho_{\mathrm{s}}, \mu_{\mathrm{f}}, D_{\mathrm{f}}, \lambda_{\mathrm{f}}). \tag{9}$$

The application of the π-theorem to (9) leads to the relation

$$E' = \varphi\left(Re_{\mathrm{mf}}, Pr, Sc, \frac{c_{\mathrm{s}}}{c_{\mathrm{f}}}, \frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{f}}}\right). \tag{10}$$

Using the Todes formula [14]

$$Re_{\mathrm{mf}} = \frac{Ar}{1400 + 5.22\sqrt{Ar}}, \tag{11}$$

it can be written instead of (10)

$$E' = \varphi\left(Ar, Pr, Sc, \frac{c_{\mathrm{s}}}{c_{\mathrm{f}}}, \frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{f}}}\right). \tag{12}$$

Not considering mass transfer processes and taking into account the fact that for gases Pr is variable within a rather narrow range, Eq. (12) can be simplified

$$E' = \varphi\left(Ar, \frac{c_{\mathrm{s}}}{c_{\mathrm{f}}}, \frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{f}}}\right). \tag{13}$$

Thus, the emulsion phase is characterized by three governing dimensionless parameters

$$Ar, \frac{c_{\mathrm{s}}}{c_{\mathrm{f}}}, \frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{f}}}. \tag{14}$$

A principal difference between linear and velocity scales in the bubble (8) and emulsion (14) phases is noted: in the bubble phase—H_{mf}, $u - u_{\mathrm{mf}}$; in the emulsion phase—d, u_{mf}.

2.3. Fluidized bed

The consequence of two-dimensionality of a FB is the fact that in a general case the transport characteristic of the bed FB is determined by the superposition (7) and (13)

$$(FB)' = \Psi\left(Fr, \frac{H_{\mathrm{mf}}}{D}, \frac{h}{H_{\mathrm{mf}}}, Ar, \frac{c_{\mathrm{s}}}{c_{\mathrm{f}}}, \frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{f}}}\right). \tag{15}$$

Thus, there are six governing dimensionless parameters characterizing local similarity of a FB (similarity of local transport processes in a FB):

$$Fr, \frac{H_{\mathrm{mf}}}{D}, \frac{h}{H_{\mathrm{mf}}}, Ar, \frac{c_{\mathrm{s}}}{c_{\mathrm{f}}}, \frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{f}}}, \tag{16}$$

The equality of five dimensionless criteria

$$Fr, \frac{H_{\mathrm{mf}}}{D}, Ar, \frac{c_{\mathrm{s}}}{c_{\mathrm{f}}}, \frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{f}}} \tag{17}$$

allows one to speak of the integral similarity of a FB (similarity of integral characteristics of a FB).[1]

3. Similarity of hydrodynamic and heat and mass transfer processes. Examples

In this part of the paper the similarity of transport processes will be considered in view of generalization of experimental data in the form of dependences of a specific determinable dimensionless characteristic of a FB on the governing criteria (16) and (17).

3.1. Dimensions of gas bubbles

As is known, a complex collective motion of gas bubbles which grow and accelerate with ascending occurs in a bubble fluidized bed.

Presented here are three most substantiated and practically tested formulas for determining a local size of bubbles in the case of porous distribution of gas when the entrance effect (the influence of an initial bubble size) can be neglected

$$D_{\mathrm{h}} = 0.42(u - u_{\mathrm{mf}})^{2/5} h^{4/5}/g^{1/5}, \quad [15], \tag{18}$$

$$D_{\mathrm{h}} = 0.78(u - u_{\mathrm{mf}})^{1/2} h^{3/4}/g^{1/4}, \quad [16], \tag{19}$$

$$D_{\mathrm{h}} = 1.30(u - u_{\mathrm{mf}})^{2/3} h^{2/3}/g^{1/3}, \quad [17]. \tag{20}$$

As can be easily shown, the dimensionless forms of (18)–(20) are

$$D_{\mathrm{h}}/h = 0.42 Fr^{1/5} (h/H_{\mathrm{mf}})^{-1/5}, \tag{21}$$

$$D_{\mathrm{h}}/h = 0.78 Fr^{1/4} (h/H_{\mathrm{mf}})^{-1/4}, \tag{22}$$

$$D_{\mathrm{h}}/h = 1.30 Fr^{1/3} (h/H_{\mathrm{mf}})^{-1/3}. \tag{23}$$

The use of the local number $Fr_{\mathrm{h}} = ((u - u_{\mathrm{mf}})^2/gh)$ simplifies these formulas even more

$$D_{\mathrm{h}}/h = 0.42 Fr_{\mathrm{h}}^{1/5}, \tag{24}$$

[1] In what follows it is shown that to describe similarity of heat and mass transfer of a FB with submerged surfaces the number Re should be included into (16) and (17).

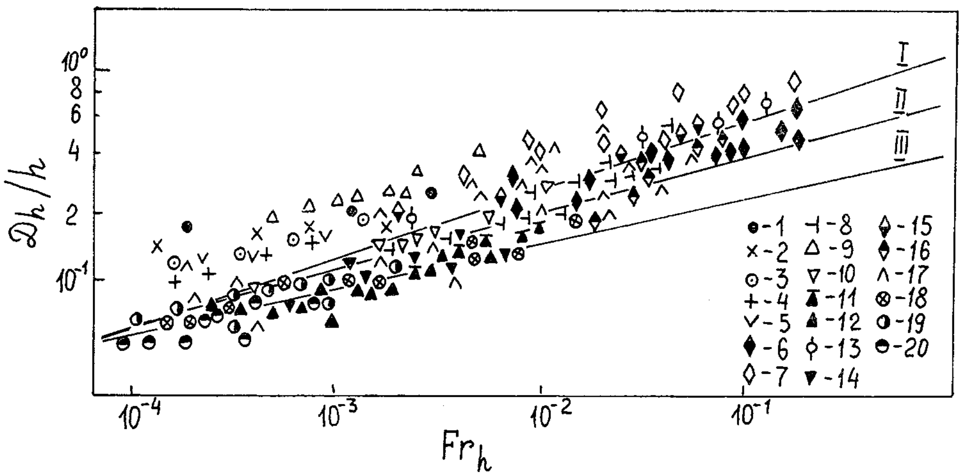

Fig. 1. Vertical size of gas bubbles. 1–5, [18]; 6–8, [19]; 9, [20]; 10, [21]; 11, [22]; 12, [23]; 13, [19]; 14, [23]; 14, [13]; 16, [24]; 17, [25]; 18–19, [23]. I, calculation by (26); II, (25); III, (24).

$$D_h/h = 0.78 Fr_h^{1/4}, \tag{25}$$

$$D_h/h = 1.30 Fr_h^{1/3}, \tag{26}$$

Fig. 1 presents the comparison of numerous test data [13,18–25] and the values of D_h calculated by (24)–(26). As is seen, Eq. (26) is the best to describe experimental data within the entire range of variation of experimental conditions.

Fig. 2 gives the results of the generalization, by the suggested technique, of the test data obtained by Podberezskiy and Rybchinskiy on the measurement of gas bubble dimensions in beds of millet particles under pressure. The obtained relation turned to be close to the formula of Rowe (25)

$$D_h/h = 0.80 Fr_h^{0.28}. \tag{25a}$$

3.2. Bed expansion

This phenomenon is closely connected to the character of a gas bubble flow in the bed and the value of FB expansion reflects concentration of bubbles in the bed volume. Consequently, bed expansion can be described by the correlation

$$\frac{H}{H_{mf}} - 1 = f\left(Fr, \frac{H_{mf}}{D}\right) \tag{27}$$

which follows from (7). Processing of numerous experimental data [13,24,26–34] yielded the following extremely simple and universal relations:

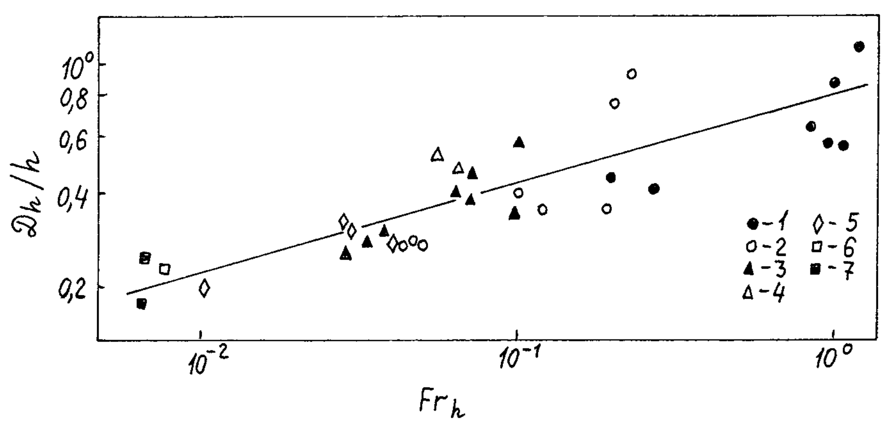

Fig. 2. Vertical size of gas bubbles in beds under pressure. 1–7, $P = 0.1$; 0.6; 1.1; 1.6; 2.1; 2.6; 3.6 MPa. Solid line is constructed by Eq. (25a).

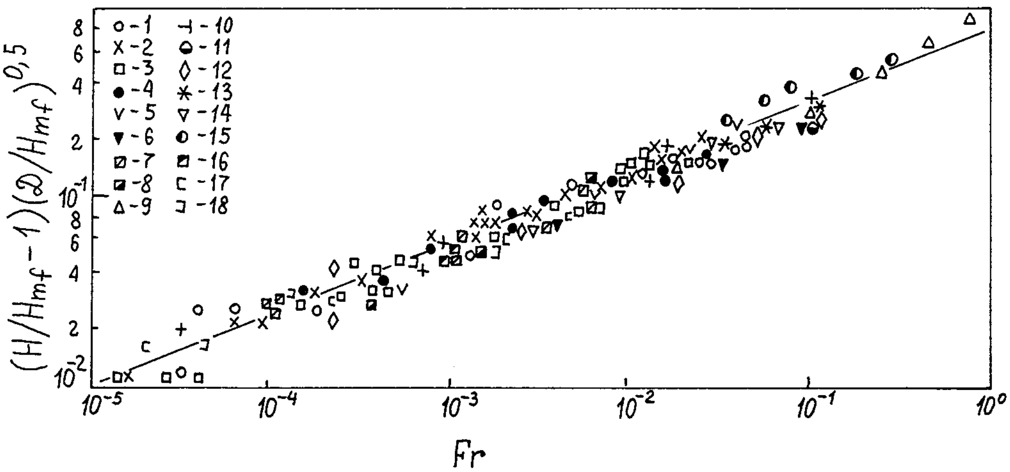

Fig. 3. Expansion of fine-particle fluidized beds [26]. 1–4, [26]; 5, [27]; 6, [28]; 7, 8, [29]; 9, [30]; 10, 11, [31]; 12–14, [32]; 15, [33]; 16, [34]; 17, 18, [26]. Solid line is constructed by Eq. (28).

(a) fine particles ($0.074 \times 10^{-3} \leq d \leq 0.38 \times 10^{-3}$ m) [26]:

$$\frac{H}{H_{\mathrm{mf}}} - 1 = 0.70 Fr^{1/3} \left(\frac{H_{\mathrm{mf}}}{D} \right)^{1/2}, \tag{28}$$

(b) coarse particles ($d = (1-2) \times 10^{-3}$ m) [35]:

$$\frac{H}{H_{\mathrm{mf}}} - 1 = 0.54 Fr^{0.54}, \tag{29}$$

shown in Figs. 3 and 4.

3.3. Bubble frequency

Frequency of gas bubble passage at the given point of a FB reflects the laws of their collective motion. In spite of relative stability of this quantity, it, nevertheless, depends on a number of factors. In [36] we obtained the following dimensionless relation which generalizes the available test data

$$\frac{f_{\mathrm{b}} H_{\mathrm{mf}}}{u - u_{\mathrm{mf}}} = 1.62 \left(\frac{Ar}{1400 + 5.22\sqrt{Ar}} \right)^{0.50} Fr^{-0.39} \left(\frac{h}{H_{\mathrm{mf}}} \right)^{-0.39}. \tag{30}$$

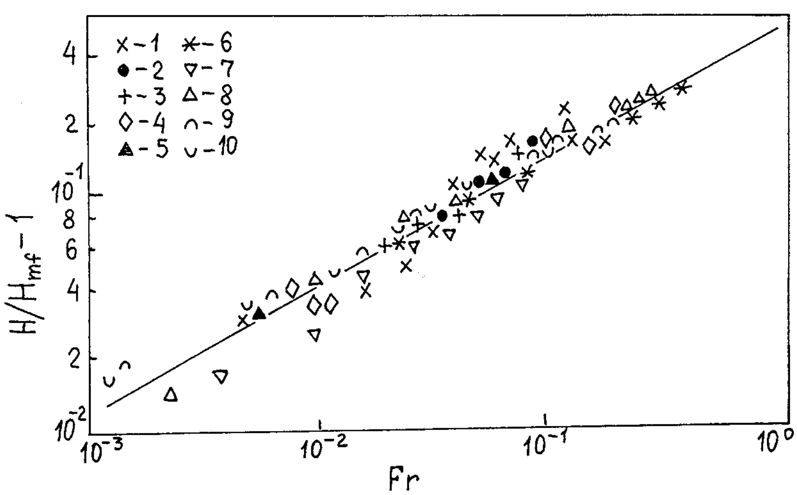

Fig. 4. Expansion of coarse-particle fluidized beds [35]. 1–5, [24]; 6–10, [13]. Solid line is constructed by Eq. (29).

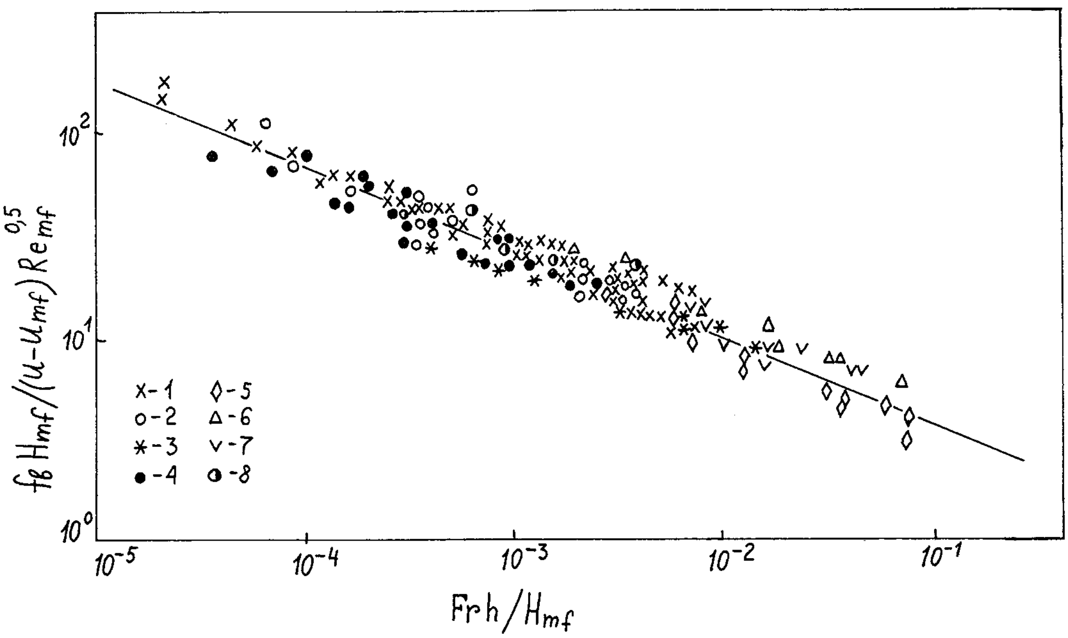

Fig. 5. Gas bubbles frequency in a fluidized bed [35]. 1, [37]; 2, [38]; 3, [39]; 4, [40]; 5–7, [36]; 8, [41]. Solid line is constructed by Eq. (39).

Fig. 5 shows the results of the comparison of test [37–41] and calculated values of f_b.

3.4. Mass exchange between a bubble and the emulsion phase

In [42] it was obtained for the surface coefficient of mass exchange between a single bubble and the emulsion phase when $Ar > 500$

$$Sh = 6.7 \times 10^4 (Pe_b Re_b)^{1/2} Ar^{1/2}. \tag{31}$$

Using an obvious relation between the surface (k) and volumetric (β) coefficients of mass transfer

$$\beta = \frac{6\varepsilon_b}{\langle D_b \rangle} k, \tag{32}$$

it is obtained on the basis of Eq. (31), relation of the 'ideal' two-phase theory (2) and a preliminary averaged

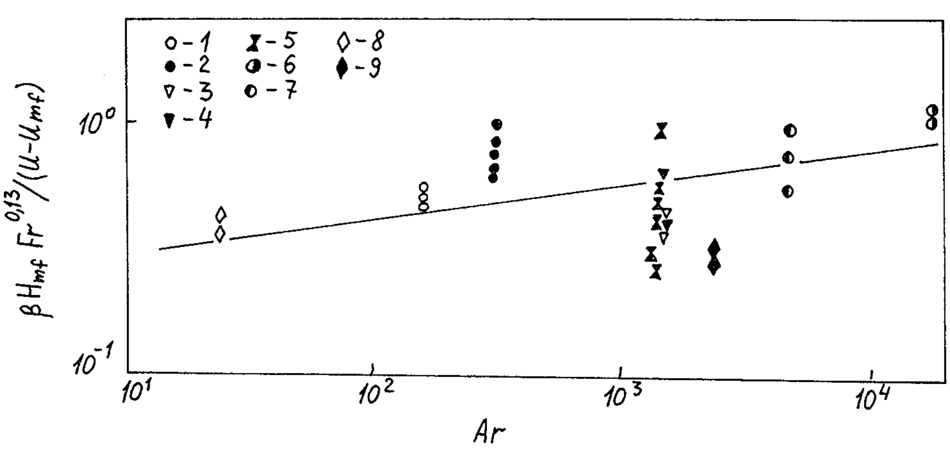

Fig. 6. Coefficient of mass transfer between bubbles and an emulsion phase. 1, [45]; 2–4, [44]; 5, [42]; 6–9, [43]. Solid line is constructed by Eq. (34).

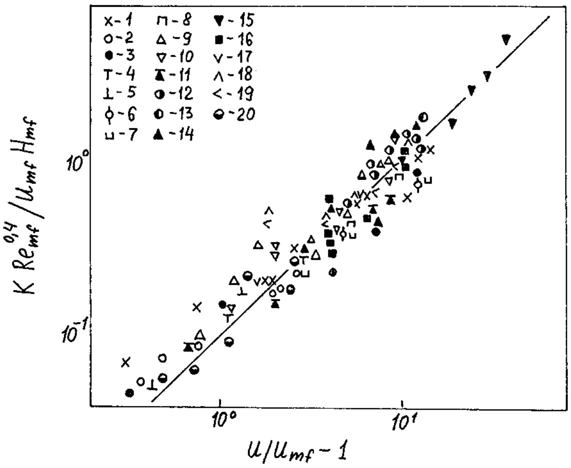

Fig. 7. Coefficient of vertical effective diffusion of particles in a fluidized bed [11]. 1–11, [11]; 12, [46]; 13, [34]; 14, 15, [47]; 16, [48]; 17–19, [49]; 20, [50]. Solid line is constructed by Eq. (36).

relation (26)

$$\frac{\beta H_{\text{mf}}}{u-u_{\text{mf}}}=k_0 Fr^{-1/3}Ar^{1/2}. \tag{33}$$

Processing of experimental data [42–45] by the values of β refined the dependences on Fr and Ar and resulted in the following simple formula

$$\frac{\beta H_{\text{mf}}}{u-u_{\text{mf}}}=0.21 Fr^{-0.13}Ar^{0.14}, \tag{34}$$

which corresponds the test data with a root-mean-square error of 18% (Fig. 6).

3.5. Solid mixing

As is known, extremely large values of the coefficients of thermal conductivity and diffusion of FB particles are due, first of all, to the presence of powerful convective flows of particles in the bed that are caused by passage of gas bubbles through the bed. In [11] the following model relation for the coefficient of vertical effective diffusion of particles is suggested

$$K=q\alpha_{\text{b}}\varepsilon_{\text{b}}D_{\text{b}}u_{\text{b}}, \tag{35}$$

where α_{b} is the relative volume of the gas bubble wake. On the basis of (35) with the employment of earlier obtained results on test data correlation over the characteristics of gas bubbles in [11] the formula for calculating K was obtained

$$\frac{K}{(u-u_{\text{mf}})H_{\text{mf}}}=0.1\left(\frac{Ar}{1400+5.22\sqrt{Ar}}\right)^{-0.4}, \tag{36}$$

which generalizes a large amount of data [11,46–50] and is shown in Fig. 7. In [5] a similar formula for the coefficient of horizontal diffusion of particles was obtained

$$\frac{D^{\text{h}}}{(u-u_{\text{mf}})H_{\text{mf}}}=0.013 Fr^{-0.15}\left(\frac{D}{H_{\text{mf}}}\right)^{0.5}, \tag{37}$$

which correlates the data of [50–56] and is shown in Fig. 8.

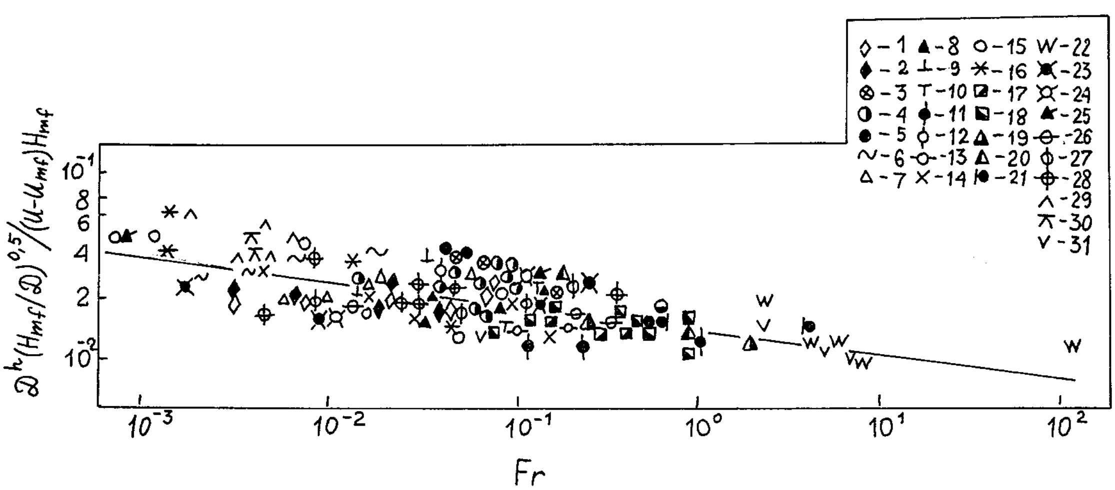

Fig. 8. Coefficient of horizontal diffusion of particles in a fluidized bed [51]. 1, 2, [52]; 3–5, [50]; 6–10, [53]; 11–13, [54]; 14–20, [51]; 21, 22, [55]; 23–30, [51]; 31, [56]. Solid line is constructed by Eq. (37).

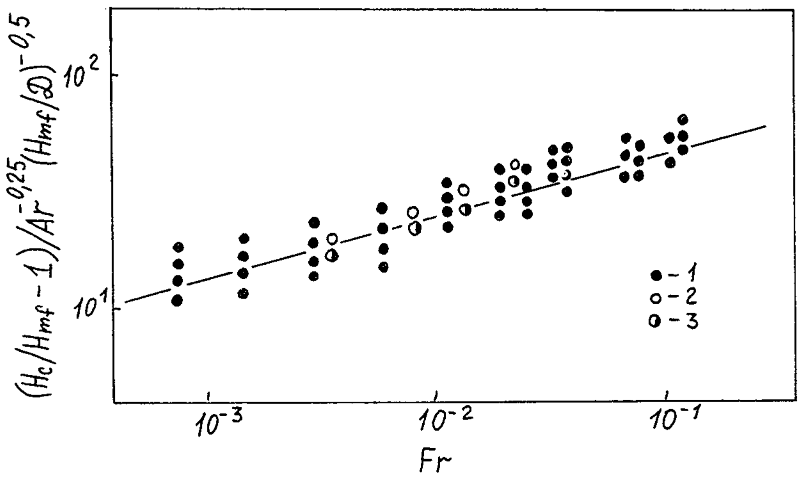

Fig. 9. Height of the separation zone of a fine-particle fluidized bed. 1, $d=0.317$ mm; 2, 3, $d=0.134$ and 0.229 mm [57]. Solid line is constructed by Eq. (39).

3.6. Height of a separation zone of a FB

A specific feature of a bubbling fluidized bed is the presence of a rather large freeboard, where particles are carried away from the bed. Entrainment of particles is formed by gas bubbles escaping from the bed and forming gas flow nonuniformities on the FB surface. Test data of [57] for small corundum particles with $d=0.134$, 0.229, and 0.317 mm were correlated by a system of dimensionless criteria

$$Fr,\ \frac{H_{\rm mf}}{D},\ Ar \tag{38}$$

which follows from (17) if the effect of c_s/c_f, ρ_s/ρ_f is neglected in it. To calculate H_c we obtained the follow-

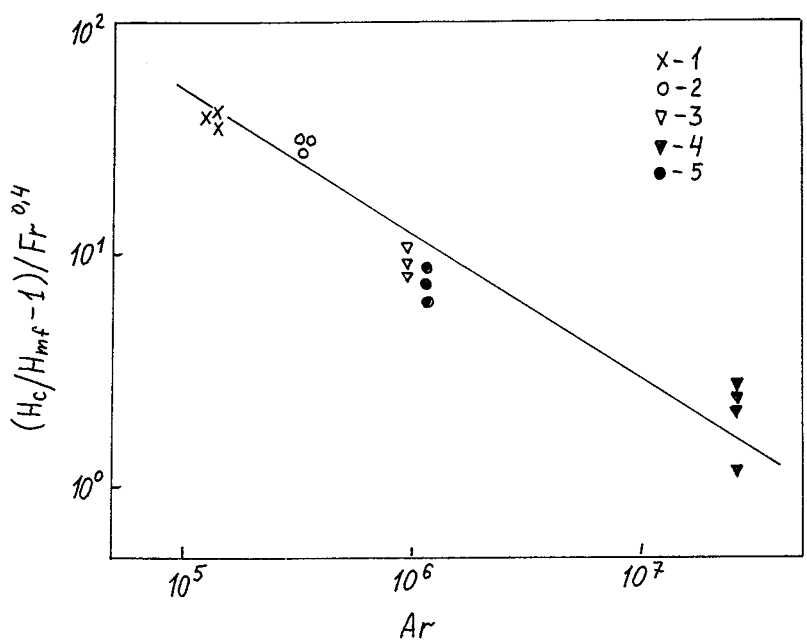

Fig. 10. Height of the separation zone of a coarse-particle fluidized bed. 1, $d=0.967$; 2, 1.33; 3, 1.85; 4, 5.6 mm [57]; 5, $d=3.16$ mm [58]. Solid line is constructed by Eq. (40).

ing simple relation

$$\frac{H_c}{H_{mf}} - 1 = 82\left(\frac{Fr}{Ar}\right)^{0.25}\left(\frac{H_{mf}}{D}\right)^{-0.5}, \tag{39}$$

shown in Fig. 9. Processing of experimental results [57,58] in the beds of coarse particles $d = 0.967$, 1.33, 1.85, 3.16, and 5.6 mm made it possible to find an analogous relation

$$\frac{H_c}{H_{mf}} - 1 = 4.8 \times 10^4 Fr^{0.4} Ar^{-0.6}, \tag{40}$$

given in Fig. 10. With allowance for the above statements about the formation of particle entrainment from a FB we can conclude that the presence of Fr number in (39) and (40) is the reflection of the objective causal-effective relationship between the phenomena of gas bubble coming to the bed surface and particle entrainment into the freeboard.

3.7. Heat exchange between a FB and the surface

Due to the complexity of the phenomenon (heat transfer with the surface is performed by both gas and particles) one did not succeed in obtaining simple one-term relations for calculating the coefficient of conductive–convective heat transfer within a rather wide range of variation of the test conditions. To describe the phenomenon, a number of models were suggested, see, e.g. the survey in [59]. The most physically substantiated model, which allows for a two-phase nature of the system, was and remains the Mickley and Fairbanks packet model [60]. In accordance with this model [59][2]

$$\alpha_{c-c} = \frac{1-\varepsilon}{1-\varepsilon_{mf}}\alpha_e + \alpha_{conv}, \tag{41}$$

where ε is the mean bed porosity; α_e is the coefficient of heat transfer with the particles (aggregates) of the emulsion phase; α_{conv} is the coefficient of heat transfer with gas passing through a FB. On the basis of the system of dimensionless parameters for the emulsion phase (14) it can be written

$$Nu_e = \frac{\alpha_e d}{\lambda_f} = f\left(Ar, \frac{c_s}{c_f}, \frac{\rho_s}{\rho_f}\right). \tag{42}$$

Assuming (42) to be power-law, we obtain on the basis of (41) for the conductive component of the coefficient of heat transfer

$$Nu_{cond} = \frac{1-\varepsilon}{1-\varepsilon_{mf}} Nu_e = A\, Ar^a \left(\frac{c_s}{c_f}\right)^b \left(\frac{\rho_s}{\rho_f}\right)^c (1-\varepsilon), \tag{43}$$

where A, a, b, c are the dimensionless coefficients (due to the fact that $1-\varepsilon_{mf}$ changes, as a rule, slightly, this quantity is introduced into A).

As follows from the structure of formula (41), the convective component of the coefficient α_{c-c} is determined by a full amount of gas filtering through the bed, i.e. by the superficial gas velocity u. Within the framework of the similarity theory this means that the dimensionless coefficient $\alpha_{conv} d/\lambda_f$ will be determined by an additional parameter, i.e. the Reynolds number $Re = u d \rho_f/\mu_f$.[3] One of the latest and the most universal formulas for calculating α_{c-c} is obtained in [61] and has the form

$$Nu_{c-c} = 0.74 Ar^{0.1}\left(\frac{c_s}{c_f}\right)^{0.24}\left(\frac{\rho_s}{\rho_f}\right)^{0.14}(1-\varepsilon)^{2/3} + 0.046 Re\, Pr(1-\varepsilon)^{2/3}/\varepsilon, \tag{44}$$

($0.1 \times 10^{-3} \le d \le 4.0 \times 10^{-3}$ m; $0.1 \le P \le 10.0$ MPa; $1.4 \times 10^2 \le Ar \le 1.1 \times 10^7$). It can be easily seen that the structure of (44) agrees with (41) and (43) and Re is among the parameters determining the convective component α_{c-c}

Allowing for the appearance of Re in the system of determining dimensionless parameters, using (17) as a basis, a complete system of governing dimensionless parameters can be presented

$$Fr, Ar, Re, \frac{H_{mf}}{D}, \frac{c_s}{c_f}, \frac{\rho_s}{\rho_f}. \tag{45}$$

The equality of similar numbers in (45) for two different FB's indicates, apparently, their full geometrical, hydrodynamic, and thermal similarity (equality of corresponding dimensionless integral characteristics of a FB).

Complete local similarity of two different FBs is realized in the case of the corresponding equality of seven governing parameters

$$Fr, Ar, Re, \frac{H_{mf}}{D}, \frac{c_s}{c_f}, \frac{\rho_s}{\rho_f}, \frac{h}{H_{mf}}. \tag{46}$$

In this case the corresponding dimensionless local characteristics of a FB will be equal. Below (see Table 1) specific examples of FB similarity are presented for

[2] It is assumed that the coefficient of heat transfer between the surface and a gas bubble is close to the coefficient of heat transfer between the surface and gas filtering in the emulsion phase.

[3] The known analogy between convective heat transfer and mass transfer makes it possible to conclude that the dimensionless coefficient of mass transfer will also be determined by Re.

Table 1
Similarity of fluidized beds

Quantity	FB furnace (FB1)	Cold FB (FB2)	FB gas generator (FB3)	Cold FB (FB4)
D, m	1	0.18	1	1.62
H_{mf}, m	1	0.18	1	1.62
u, m/s	2	0.92	2	2.52
$2u_{mf}$, m/s	1	0.50	0.49	0.61
d, m	2.5×10^{-3}	0.55×10^{-3}	2×10^{-3}	3.9×10^{-3}
μ_f, kg/m s	449×10^{-7}	179×10^{-7}	365×10^{-7}	179×10^{-7}
ρ_f, kg/m^3	0.31	1.2	4.81	1.2
ρ_s kg/m^3	2000	7741	2000	500
	Similarity		Similarity	

the cases simulating a furnace ($T=800°$C, $P = 0.1$ MPa) and a gas generator ($T=1000°$C, $P = 2$ MPa) with a FB. Cold FBs are the beds fluidized by air at $T=20°$C and $P = 0.1$ MPa. The bed FB1 is completely similar to FB2, and FB3 to FB4 because they have equal corresponding parameters in (45). We note that the similarity by the simplex c_s/c_f cannot be obtained due to indirect relation between c and ρ.

3.8. Coefficient of heat transfer and particle concentration in a FB freeboard

Heat transfer in this zone is determined, as known [14], by the concentration of particles which are thrown from the bed in the wakes of gas bubbles. The wake velocity assigns initial velocity of particles on the upper boundary of the bed and determines distribution of their concentration over the height of the freeboard [14]

$$\rho = \rho_{su}\ \exp\left(-\frac{2g(h-H_{mf})}{W_{su}^2}\right). \tag{47}$$

On the basis of (47) Palchenok and Hassan [62], assuming $W_{su} \sim (V_b)_{su} = 0.711\sqrt{g(D_b)_{su}}$, found a simple relation between ρ and $(D_b)_{su}$

$$\rho = \rho_{su}\left(-\frac{2g(h-H_{mf})}{n(D_b)_{su}}\right). \tag{48}$$

Using (18) and (20), the authors of [62] obtained relations for calculating ρ:

(a) fine particles ($0.10\times10^{-3} \le d \le 0.315\times10^{-3}$ m)

$$\frac{\rho}{\rho_{su}} = \exp\left(-1.2\frac{h-H_{mf}}{H_{mf}}Fr^{-1/3}\right), \tag{49}$$

(b) coarse particles ($1.6\times10^{-3} \le d \le 4.0\times10^{-3}$ m)

$$\frac{\rho}{\rho_{su}} = \exp\left(-2.4\frac{h-H_{mf}}{H_{mf}}Fr^{-1/5}\right). \tag{50}$$

Assuming then, as in a FB (see (44)), $(\alpha_{c-c})_{fb} \sim (1-\varepsilon)^{2/3} \sim \rho^{2/3}$, on the basis of (49) and (50) the authors of [63] obtained the following formulas for calculating heat transfer in the FB freeboard:

(a) fine particles

$$A = \frac{(\alpha_{c-c})_{fb}-\alpha_0}{\alpha_{c-c}-\alpha_0} = \exp\left(-0.8\frac{h-H_{mf}}{H_{mf}}Fr^{-1/3}\right), \tag{51}$$

(b) coarse particles

$$A = \exp\left(-1.6\frac{h-H_{mf}}{H_{mf}}Fr^{-1/5}\right), \tag{52}$$

where $\alpha = 0.78\lambda_f Re_t^{0.5}/D_t$. In [64] the formula was obtained for calculating $(\alpha_{c-c})_{fb}$ in the beds of coarse particles ($1.6\times10^{-3} \le d \le 4.0\times10^{-3}$ m) which does not involve α_{c-c}

$$(Nu_{c-c})_{fb} - Nu_0 = 0.29Fr^{0.66((h-H_{mf})/H_{mf})^{1.32}} \times Ar^{0.27}\ \exp\left(-1.4\frac{h-H_{mf}}{H_{mf}}\right), \tag{53}$$

where $Nu_0 = 0.78dRe_t^{0.5}/D_t$.

4. Conclusions

The method for obtaining similarity criteria of a FB is suggested which is based on simultaneous use of the π-theorem of the similarity theory and Eq. (1) of the two-phase theory of fluidization allowing for the presence of bubble and emulsion phases in the system.

The system of seven dimensionless criteria of local similarity

$$Fr, Ar, Re, \frac{H_{mf}}{D}, \frac{c_s}{c_f}, \frac{\rho_s}{\rho_f}, \frac{h}{H_{mf}}.$$

is found.

The equality of the corresponding quantities of this system for two different FBs makes it possible to speak of their full local similarity (equality of all dimensionless local and integral characteristics).

The system of six dimensionless criteria

$$Fr, Ar, Re, \frac{H_{\mathrm{mf}}}{D}, \frac{c_{\mathrm{s}}}{c_{\mathrm{f}}}, \frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{f}}}$$

allows one to determine integral similarity of FBs. Their corresponding equality for two FBs indicates full integral similarity (equality of all dimensionless integral geometric, hydrodynamic, and heat transfer characteristics).

The correlations obtained on the basis of the developed method of the similarity theory for calculating local and integral characteristics of a FB are a particular expression of the similarity of transport processes in the system. They are distinguished by simplicity, physical and large universal nature and are convenient for practical use. The employment of the present technique makes the work on experiment planning and on generalization of the obtained experimental data considerably simpler.

In conclusion it is noted that the formulated approach is transferred without difficulty to packed FBs where additional limitations are imposed on gas bubble motion that are caused by the presence of a fixed packing, e.g. a tube bundle, in the bed. This, consequently, leads only to the appearance of additional quantities (ε_{p}, D_{t}, S_{v}, S_{h}) in (6) which characterize the packings. In this case also introduced is a modified Froude number $Fr^* = (u/\varepsilon_{\mathrm{p}} - u_{\mathrm{mf}})^2/gS_{\mathrm{v}}$, which characterizes gas bubble motion in the intertube space instead of $Fr = (u - u_{\mathrm{mf}})^2/gH_{\mathrm{mf}}$ [65].

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